Energy of a Static Magnetic Field

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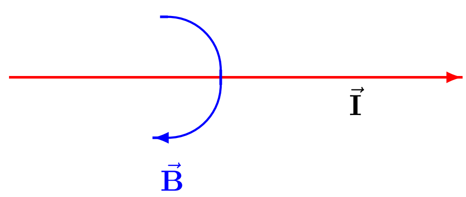
A magnetic field B can be created by a current I flowing through a straight wire, as depicted below.



If you put the thumb of your right hand in the direction of the current $\vec{\mathbf{I}}$, then your fingers will naturally curl in the direction of the magnetic field $\vec{\mathbf{B}}$. In a vacuum or air, the dependence of the magnitude of the magnetic field, B , at a distance r from the wire upon the current and r is

$$B = \frac{\mu_o I}{2\pi r}, \tag{1}$$

where $\mu_o = 4\pi \times 10^{-7} Tm/A$ is the permeability of the vacuum or free space. The current is measured in $Amperes(A) = Coulomb(C)/s$, and the magnetic field is measured in $Tesla(T)$. If the wire is in the form of a circular loop, then the magnetic field at the center of the loop points in the direction of the thumb of your right hand when your fingers curl in the direction of the current and has the magnitude

$$B = \frac{\mu_o I}{2R}, \tag{2}$$

where R is the radius of the loop. Now, if the wire is wrapped into a cylindrical coil of length d and cross sectional area σ with n turns or loops per meter of coil length, the magnetic field is found to have the same magnitude everywhere within the coil,

$$B = \mu_o I n. \tag{3}$$

The direction of $\vec{\mathbf{B}}$ is in the direction that the thumb of the right hand points when the fingers curl in the direction of the current within the loops of the coil, and it is also the same everywhere within the coil. The coil also has a property called inductance, which is given by the relation

$$L = \mu_o n^2 \sigma d = \mu_o n^2 \times volume. \tag{4}$$

The unit of inductance is the $Henry(H) = Tm^2/A$. The energy stored within the coil is

$$U = \frac{1}{2} L I^2, \tag{5}$$

with the units here being $J = HA^2$. This energy can also be written in terms of B ,

$$U = \frac{1}{2} L I^2 = \frac{1}{2} L (\frac{B}{n\mu_o})_2 = \frac{1}{2} \mu_o n^2 \sigma d (\frac{B}{n\mu_o})_2 = \frac{1}{2} \frac{B^2}{\mu_o} \times volume. \tag{6}$$

Thus, the energy density of a static magnetic field is

$$u = \frac{1}{2} \frac{B^2}{\mu_o}. \tag{7}$$

When the vacuum within the coil is filled with a material, μ_o is replaced with $\mu = \mu_o K_m$, where K_m is called the permeability constant.

The permeability constant of a material is a measure of how magnetically polarizable it is, that is, how large on the average a magnetic dipole field can be induced within all the microscopic regions of the material. The magnetic field $\vec{\mathbf{B}}$ within the coil above creates the magnetization $\vec{\mathbf{M}}$, or magnetic dipole moment per unit volume within the permeable material, through the relation

$$\vec{\mathbf{M}} = \frac{\chi_m}{1 + \chi_m} \vec{\mathbf{B}}. \tag{8}$$

The permeability constant K_m is derived from the susceptibility χ_m through

$$K_m = 1 + \chi_m. \tag{9}$$

Material	Permeability constant K_m for static field
vacuum	1
air	1